Appendix: Revised Simos

A.1 Introduction

An important and challenging step in the solution of a decision making problem is the elicitation of weights. In the examples presented in the previous Chapters, the weights of the criteria were inputs to the problem. However, one of the most important steps that the decision maker performs during the solution of a decision making problem with an MCDA method is the assessment of the criteria weights. Various methods have been proposed for this task. These methods can be mainly categorized in two major classes, the direct assessment ones and the indirect ones. The indirect assessment methods have been used in most applications of MCDA methods due to their simplicity and realism. One of the most widely-used methods is the one proposed by Simos [2, 3]. This method is a typical indirect assessment method that has been widely-used in decision making problems since it is relatively easy for decision makers to express their preferences. The elicitation of the criteria weights is performed by asking decision makers to express the relative importance of the criteria, through the arrangement of criteria cards, from the least to the most important one. The method was later extended by Figuera and Roy [1] in order to address certain robustness issues of the original method. The revised Simos method is widely-used in decision making problems for estimating the criteria weights.

A.2 Methodology

The decision maker is given a set of cards. The name of each criterion is written on a card. The decision maker uses the cards in order to rank the criteria from the least important to the most important. The first criterion in the ranking is the least important and the last criterion is the most important. If some criteria have the same importance for the decision maker, he/she can place them together in the same

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position. Therefore, a complete pre-order of the whole n criteria is obtained. The number of ranks is $\widetilde{n}$, where $1 \leq \widetilde{n} \leq n$ (since some of the cards can be placed in the same rank).

The decision maker has also a set of white cards. The importance of two successive criteria (or two successive subsets of ex aequo criteria in case two or more cards have been placed together) in the ranking can be more or less close. In order to depict this smaller or larger difference of the importance of successive criteria, the decision maker introduces white cards between two successive cards. The more the number of white cards between two successive criteria, the greater the difference between their importance. If no white card is placed between two successive ranks, then the difference between the weights of the criteria in these two successive ranks can be chosen as the unit, u, for measuring the intervals between weights. Hence, if one white card is placed between two successive ranks, then there is a difference of $2u$ between the weights of the criteria in these two successive ranks. Finally, the decision maker should state how many times the last criterion is more important than the first one. This ratio is denoted by the parameter z.

The revised Simos method consists of two steps:

1. Calculate the non-normalized weights $k = (k_1, k_2, \cdots, k_{\widetilde{n}})$: Let ${e_r}'$ be the number of white cards between the ranks r and $r+1$.

$$\begin{cases} e_r = {e_r}' + 1, \forall r = 1, 2, \cdots, \widetilde{n} - 1, {e_1}' = 0 \\ u = \frac{z-1}{\sum_{r=1}^{\widetilde{n}-1} e_i} \end{cases} \tag{A.1}$$

   Next, we can calculate the non-normalized weights k as follows:

$$k_r = 1 + u \sum_{i=0}^{r-1} e_i, e_0 = 0 \tag{A.2}$$

2. Calculate the normalized weights $k^* = \left(k_1^*, k_2^*, \cdots, k_{\widetilde{n}}^*\right)$: Let c_i be the number of cards in each ranking i, where $1 \leq i \leq \widetilde{n}$. Then, the normalized weights k^* are calculated as follows:

$$k_r^* = \frac{100}{\sum_{i=1}^{\widetilde{n}} c_i k_i} k_r \tag{A.3}$$

Figuera and Roy [1] also presented a method to eliminate some of the decimal figures from the normalized weights.

A.2.1 Numerical Example

Let us consider a set of eight criteria: $\{a, b, c, d, e, f, g, h\}$. Let us also suppose that the decision maker groups together cards associated to the criteria having the same importance into four subsets of ex aequo, inserting also three white cards between some of the successive ranks. Table A.1 presents the ranking of these cards.

Let us suppose that $z = 6.5$, i.e., the decision maker states that the last subset is 6.5 times more important than the first one. We start by calculating vector e and parameter u according to Equation (A.1)

$$e = \begin{bmatrix} 1 & 2 & 3 & 1 \end{bmatrix}$$

$$u = 1.375$$

Then, we can calculate the non-normalized weights k according to Equation (A.2)

$$k = \begin{bmatrix} 1 & 2.375 & 5.125 & 9.25 \end{bmatrix}$$

Finally, we calculate the normalized weights k^* according to Equation (A.3)

$$k^* = \begin{bmatrix} 2.61437908 & 6.20915033 & 13.39869281 & 24.18300654 \end{bmatrix}$$

Therefore, the criteria weights are presented in Table A.2.

Table A.1 Ranking of the criteria using cards

Rank	Subset of ex aequo
1	$\{b, d\}$
2	$\{c\}$
3	White card
4	$\{e, f, h\}$
5	White card
6	White card
7	$\{a, g\}$

Table A.2 Criteria weights

Criterion	Weight
a	24.1830065359
b	2.61437908497
c	6.2091503268
d	2.61437908497
e	13.3986928105
f	13.3986928105
g	24.1830065359
h	13.3986928105
Total	100

A.2.2 Python Implementation

The file *RevisedSimos.py* includes a Python implementation of the revised Simos method. The input variables are array *subsets* (the ranks of the criteria, lines 8–9), and z (the parameter z, line 12). In lines 15–22, the number of cards, the number of positions with non-white cards, and vector c are calculated. Then, we calculate parameter u (line 25). Next, we calculate vector e (lines 29–34). The non-normalized weights are computed in lines 37–41, while the normalized weights are computed in lines 44–46. Finally, the criteria weights are printed in lines 49–57.

```
1.  # Filename: RevisedSimos.py
2.  # Description: Revised Simos method
3.  # Authors: Papathanasiou, J. & Ploskas, N.
4.
5.  from numpy import *
6.
7.  # placement of cards ('w' represents a white card)
8.  subsets = array([['b','d'], ['c'], ['w'],
9.     ['e', 'f', 'h'], ['w'], ['w'], ['a', 'g']])
10.
11. # parameter z
12. z = 6.5
13.
14. # calculate number of cards, positions, and vector c
15. noOfcards = 0
16. positions = 0
17. c = []
18. for i in range(subsets.shape[0]):
19.     if subsets[i][0] != 'w':
20.         noOfcards = noOfcards + len(subsets[i][:])
21.         positions = positions + 1
22.         c.append(len(subsets[i][:]))
23.
24. # calculate u
25. U = round((z - 1) / positions, 6)
26.
```

```
27.  # calculate vector e
28.  e = ones(positions)
29.  counter = -1
30.  for i in range(subsets.shape[0]):
31.      if subsets[i][0] != 'w':
32.          counter = counter + 1
33.      else:
34.          e[counter] = e[counter] + 1
35.
36.  # calculate the non-normalized weights k
37.  k = ones(positions)
38.  totalk = k[0] * c[0]
39.  for i in range(1, positions):
40.      k[i] = 1 + U * sum(e[0:i])
41.      totalk = totalk + k[i] * c[i]
42.
43.  # calculate the normalized weights
44.  normalizedWeights = zeros(positions)
45.  for i in range(0, positions):
46.      normalizedWeights[i] = (100 / totalk) * k[i]
47.
48.  # print the criteria weights
49.  counter = -1
50.  for i in range(subsets.shape[0]):
51.      if subsets[i][0] != 'w':
52.          counter = counter + 1
53.      else:
54.          continue
55.      for j in range(len(subsets[i][:])):
56.          print("Weight of criterion ", subsets[i][j],
57.              " = ", normalizedWeights[counter])
```

References

1. Figueira, J., & Roy, B. (2002). Determining the weights of criteria in the ELECTRE type methods with a revised Simos' procedure. *European Journal of Operational Research, 139*(2), 317–326.
2. Simos, J. (1990). *Evaluer l'impact sur l'environnement: Une approche originale par l'analyse multicritère et la négociation*. Lausanne: Presses polytechniques et universitaires romandes.
3. Simos, J. (1990). *L'évaluation environnementale: Un processus cognitif négocié*. Ph.D. dissertation, DGF-EPFL, Lausanne, France.

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